## Ephemerides of the Sun

The sun is probably the most frequently observed body in celestial navigation. Greenwich hour angle and declination of the sun as well as $\mathrm{GHA}_{\text {Aries }}$ and EoT can be calculated using the algorithms listed below [8,17]. The formulas are relatively simple and useful for navigational calculations with programmable pocket calculators ( 10 digits recommended).

First, the time variable, T , has to be calculated from year, month, and day. T is the time, measured in days and dayfractions, before or after Jan 1, 2000, 12:00:00 UT:

$$
T=367 \cdot y-\text { floor }\left\{1.75 \cdot\left[y+\text { floor }\left(\frac{m+9}{12}\right)\right]\right\}+\text { floor }\left(275 \cdot \frac{m}{9}\right)+d+\frac{U T[h]}{24}-730531.5
$$

y is the number of the year ( 4 digits), m is the number of the month, and d the number of the day in the respective month. UT is Universal Time in decimal format (e.g., $12 \mathrm{~h} 30 \mathrm{~m} 45 \mathrm{~s}=12.5125$ ). For May 17, 1999, 12:30:45 UT, for example, T is -228.978646. The equation is valid from March 1, 1900 through February 28, 2100.
floor(x) is the greatest integer smaller than x . For example, floor $(3.8)=3$, floor $(-2.2)=-3$. The floor function is part of many programming languages, e.g., JavaScript. In general, it is identical with the int function used in other languages. However, there seem to be different definitions for the int function. This should be checked before programming the above formula.

Mean anomaly of the sun*:

$$
g\left[^{\circ}\right]=0.9856003 \cdot T-2.472
$$

Mean longitude of the sun ${ }^{*}$ :

$$
L_{M}\left[{ }^{\circ}\right]=0.9856474 \cdot T-79.53938
$$

True longitude of the sun*:

$$
L_{T}\left[^{\circ}\right]=L_{M}\left[{ }^{\circ}\right]+1.915 \cdot \sin g+0.02 \cdot \sin (2 \cdot g)
$$

Obliquity of the ecliptic:

$$
\varepsilon\left[{ }^{\circ}\right]=23.439-4 \cdot T \cdot 10^{-7}
$$

## Declination of the sun:

$$
\operatorname{Dec}\left[^{\circ}\right]=\arcsin \left(\sin L_{T} \cdot \sin \varepsilon\right)
$$

Right ascension of the sun (in degrees) ${ }^{*}$ :

$$
R A\left[^{\circ}\right]=2 \cdot \arctan \left(\frac{\cos \varepsilon \cdot \sin L_{T}}{\cos D e c+\cos L_{T}}\right)
$$

GHA $_{\text {Aries }}{ }^{*}$ :

$$
G H A_{\text {Aries }}\left[{ }^{\circ}\right]=0.9856474 \cdot T+15 \cdot U T[h]+100.46062
$$

## Greenwich hour angle of the sun*:

$$
G H A\left[{ }^{\circ}\right]=G H A_{\text {Aries }}\left[{ }^{\circ}\right]-R A\left[^{\circ}\right]
$$

*These quantities have to be within the range from $0^{\circ}$ to $360^{\circ}$. If necessary, add or subtract $360^{\circ}$ or multiples thereof. This can be achieved using the following algorithm which is particularly useful for programmable calculators:

$$
y=360 \cdot\left[\frac{x}{360}-\text { floor }\left(\frac{x}{360}\right)\right]
$$

## Equation of Time:

$$
G A T[h]=\frac{G H A\left[{ }^{\circ}\right]}{15}+12 h
$$

(If GAT > 24h, subtract 24h.)

$$
\operatorname{EoT}[h]=G A T[h]-U T[h]
$$

(If EoT $>+0.3 h$, subtract 24 h. If EoT $<-0.3 h$, add $24 h$.)

## Semidiameter and Horizontal Parallax

Due to the excentricity of the earth's orbit, semidiameter and horizontal parallax of the sun change periodically during the course of a year. The SD of the sun varies inversely with the distance earth-sun, R:

$$
\begin{gathered}
R[A U]=1.00014-0.01671 \cdot \cos g-0.00014 \cdot \cos (2 \cdot g) \\
\left(1 \mathrm{AU}=149.6 \cdot 10^{6} \mathrm{~km}\right) \\
S D\left[^{\prime}\right]=\frac{16.0}{R[A U]}
\end{gathered}
$$

The mean horizontal parallax of the sun is approx. 0.15 '. The periodic variation of HP is too small to be of practical significance.

## Accuracy

The maximum error of GHA and Dec is about $\pm 0.6^{\prime}$. Results have been cross-checked with Interactive Computer Ephemeris 0.51 (accurate to approx. 0.1'). Between the years 1900 and 2049, the error was smaller than $\pm 0.3^{\prime}$ in most cases ( 100 dates chosen at random). EoT was accurate to approx. $\pm 2 \mathrm{~s}$. In comparison, the maximum error of GHA and Dec extracted from the Nautical Almanac is approx. $\pm 0.25$ (for the sun) when using the interpolation tables. The error of SD is smaller than $\pm 0.1^{\prime}$.

